Another review of Kato, Fumiharu Mathematics that bridges universes. The shock of IUT theory

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Kato, Fumiharu

Mathematics that bridges universes. The shock of IUT theory. (JP) Zbl 07530203

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This book is a good introduction to the basic ideas of Shinichi Mochizuki's tetralogy on his hulking inter-universal Teichmüller theory [*S. Mochizuki*, Publ. Res. Inst. Math. Sci. 57, No. 1–2, 3–207 (2021; Zbl 1465.14002); Publ. Res. Inst. Math. Sci. 57, No. 1–2, 209–401 (2021; Zbl 1465.14003); Publ. Res. Inst. Math. Sci. 57, No. 1–2, 403–626 (2021; Zbl 1465.14004); Publ. Res. Inst. Math. Sci. 57, No. 1–2, 627–723 (2021; Zbl 1465.14005)], which is nevertheless spectacularly rickety, as is clearly observed by Peter Scholze. The book is by no means a textbook for mathematicians or graduate students in mathematics, but for general public possibly unfamiliar with modern mathematics at all. Indeed Kadokawa is a publisher specializing in books for general public. Shinichi Mochizuki has contributed a sentence to the book. Mochizuki started to attack the so-called ABC conjecture around 2000. He unveiled his tetralogy in 2012. It is at Institut Henri Poincaré in Paris in 1997 that the author first met Mochizuki. The author happened to meet Mochizuki again within the campus of Kyoto University in July 2005. From 12 July 2005 through 15 February 2011, the author and Mochizuki intimately held a seminar firstly several times a month but finally only once a month. The seminar was held at the author's then laboratory on the 7th floor of the 8th building of the faculty of sciences at Kyoto University. The seminar was held at night, after the author completed his regular work. The author and Mochizuki dined together after the seminar. Therefore the author knows well how the IUT theory has been developed. I think that, since the author's English is not bad, the book should be translated into English, which would be appreciated by mathematical community.

1. Generally speaking, the IUT theory consists of three steps.

[1] Consider a family of universes, between which local Galoir groups are communicated.

[2] Recover addition from the monoid of multiplication and the local Galois group.

[3] Measure the indefiniteness of the above recovery.

Communications on symmetry are discussed in Chapter 6, while it is argued in Chapter 5 that not a single universe but a family of universes is considered in IUT theory, multiplications in different universes being to be fitted like a jigsaw puzzle. It is argued in Chapter 7 that *anabelian geometry* [*F. Bogomolov* and *Y. Tschinkel*, Math. Sci. Res. Inst. Publ. 59, 17–63 (2012; Zbl 1290.14017); *F. Bogomolov* and *Y. Tschinkel*, Math. Sci. Res. Inst. Publ. 59, 17–63 (2012; Zbl 1290.14017)] which recovers an arithmetically geometric object from a highly nonabelian group, is exploited in recovery process in IUT theory just as, in Galois theory, the so-called Galois group tells much how to solve the given algebraic equation. Chapter 3 gives a conceptual explication of *Teichmüller theory*. The IUT theory could be regarded as Teichmüller theory developed in the context of inter-universality. The final chapter (Chapter 8) is concerned with the desired inequality

$$\deg \Theta \leqq \deg q + c$$

The origin of the IUT theory is Hodge-Arakelov theory [*S. Mochizuki*, Foundations of p-adic Teichmüller theory. Providence, RI: American Mathematical Society (1999; Zbl 0969.14013)], which holds globally, though its aspects related with the ABC conjecture are to be realized only locally.

2. The author argues once and again such and such a statement as

"While standard mathematics has developed only within a single universe, it is Shinichi Mochizuki that dealt with a family of universes for the first time."

This is completely wrong. This shows only that the author is a good bit ignorant of modern mathematics. A counter-example can be seen, e.g. in *Grothendieck toposes*, where a family of universes is considered over a category, changing continuously with respect to a Grothendieck topology [*M. Artin* (ed.) et al., Séminaire de géométrie algébrique du Bois-Marie 1963–1964. Théorie des topos et cohomologie étale des schémas (SGA 4). Un séminaire dirigé par M. Artin, A. Grothendieck, J. L. Verdier. Avec la collaboration de P. Deligne, B. Saint-Donat. Tome 3. Exposés IX à XIX. Springer, Cham (1973; Zbl 0245.00002); *M. Artin* (ed.) et al., Séminaire de géométrie algébrique du Bois-Marie 1963–1964. Théorie des topos et cohomologie étale des schémas. (SGA 4). Un séminaire dirigé par M. Artin, A. Grothendieck, J. L. Verdier. Avec la collaboration de N. Bourbaki, P. Deligne, B. Saint-Donat. Tome 1: Théorie des topos. Exposés I à IV. 2e éd. Springer, Cham (1972; Zbl 0234.00007); *M. Artin* (ed.) et al., Séminaire de géométrie algébrique du Bois-Marie 1963–1964. Théorie des topos et cohomologie étale des schémas (SGA 4). Un séminaire dirigé par M. Artin, A. Grothendieck, J. L. Verdier. Avec la collaboration de N. Bourbaki , P. Deligne, B. Saint-Donat. Tome 2. Exposes V à VIII. Springer, Cham (1972; Zbl 0237.00012); *A. Grothendieck* and *J. L. Verdier*, Lect. Notes Math. 270, 163–340 (1972; Zbl 0249.14006); *A. Grothendieck*, Lect. Notes Math. 270, 341–365 (1972; Zbl 0255.14009); *A. Grothendieck* and *J. L. Verdier*, Lect. Notes Math. 269, 299–525 (1972; Zbl 0256.18008)]. Grothendieck toposes are an avant-gardiste generalization of sheaf theory [*G. E. Bredon*, Теория пучков. Translation from the English by A. Yu. Volovikov. Translation edited by E. G. Sklyarenko (Russian). Moskva: Nauka (1988; Zbl 0638.55001)], which in turn has its origin in Kiyoshi Oka's notion of the ideal of indeterminate domains [*K. Oka*, Bull. Soc. Math. Fr. 78, 1–27 (1950; Zbl 0036.05202)]. Another can be seen in J. P. Cohen's *forcing* in the proof of independence of the continum hypothesis from ZFC (Zermelo-Fraenkel set theory with the axiom of choice) [*P. J. Cohen*, Set theory and the continuum hypothesis. New York-Amsterdam: W.A. Benjamin, Inc. (1966; Zbl 0182.01301); Proc. Natl. Acad. Sci. USA 50, 1143–1148 (1964; Zbl 0192.04401)], where a universe is enlarged to a larger universe in which the continum hypothesis fails.

3. The author gives a lot of episodes telling how nice a guy Shinichi Mochizuki is, but it is not easy to reconcile those episodes with the well-established figure of Shinichi Mochizuki in international mathematical community [The biggest mystery in mathematics: Shinichi Mochizuki and the impenetrable proof, by Davide Castelvecchi in Nature (October 2015), https://www.nature.com/articles/526178a].

"Adding to the enigma is Mochizuki himself. He has so far lectured about his work only in Japan, in Japanese, and despite being fluent in English, he has declined invitations to talk about it elsewhere. He does not speak to journalists; several requests for an interview for this story went unanswered. Mochizuki has replied to e-mails from other mathematicians and been forthcoming to colleagues who have visited him, but his only public input has been sporadic posts on his website. In December 2014, he wrote that to understand his work, there was a "need for researchers to deactivate the thought patterns that they have installed in their brains and taken for granted for so many years". To mathematician Lieven Le Bruyn of the University of Antwerp in Belgium, Mochizuki's attitude sounds defiant. "Is it just me," he wrote on his blog earlier this year, "or is Mochizuki really sticking up his middle finger to the mathematical community"."

I should say regrettably that Shinichi Mochizuki is, far from being a nice guy, mentally ill in the midst of paranoia [https://www.researchgate.net/publication/362790980_paranoia_John_Nash_and_Shinichi_Mochizuki; https://www.researchgate.net/publication/363639435_Shinichi_Mochizuki%27s_whistleblower; https://www.researchgate.net/publication/360503598_naked_genius]. I guess that Mochizuki's paranoia has affected greatly his solution of the ABC conjecture, just as John Nash's solution of the Riemann hypothesis, on which he gave a lecture at Corolado University in 1959, was affected by his paranoia [https://www.researchgate.net/publication/363613733_John_Nash%27s_solution_of_the_Riemann_conjecture_correction].

4. Mochizuki got his Ph.D. in mathematics in 1992 after completing his doctoral dissertation, titled "The geometry of the compactification of the Hurwitz scheme," under the supervision of Faltings, who won a Fields medal by settling the *Mordell conjecture* [*G. Faltings*, Invent. Math. 73, 349–366 (1983; Zbl 0588.14026)]. Faltings himself suggested the *effective Mordell conjecture* (a quantitative version of the Mordell conjecture) as a theme of Mochizuki's Ph.D. thesis (January 1991). Mochizuki did not take up this theme at that time, though the suggested theme appeared highly impressive to Mochizuki. Faltings himself does not remember at all that he really suggested such a theme to Mochizuki. It should be noted that the effective Mordell conjecture and the abc conjecture are almost equivalent. In relation to Mordell conjecture, *Diophantine equations* are discussed in Chapter 3. Mochizuki's IUT theory can be regarded as a two decades later response to the above suggested theme, though he bungled so that his Ph.D. could not be admitted.

5. The *ABC conjecture* about *ABC triples* as well as its effective version is explained in detail in Chapter 4. It is argued that some important results follow directly from the ABC conjecture or its effective version.

[1] effective Mordell conjecture

[2] Szpiro's conjecture [*L. Szpiro*, Astérisque 86, 44–78 (1981; Zbl 0517.14006); Contemp. Math. 67, 279–293 (1987; Zbl 0634.14012)]

[3] Frey conjecture [*G. Frey*, J. Reine Angew. Math. 331, 185–191 (1982; Zbl 0474.14011); Ann. Univ. Sarav., Ser. Math. 1, No. 1, 40 p. (1986; Zbl 0586.10010)]

[4] Vojta's conjecture on hyperbolic algebraic curves [*P. Vojta*, Diophantine approximations and value distribution theory. Berlin etc.: Springer-Verlag (1987; Zbl 0609.14011)]

[5] Fermat's last theorem [*A. Wiles*, Ann. Math. (2) 141, No. 3, 443–551 (1995; Zbl 0823.11029); *R. Taylor* and *A. Wiles*, Ann. Math. (2) 141, No. 3, 553–572 (1995; Zbl 0823.11030)]

6. In Chapter 1 the author protects Mochizuki's desposition to be reluctant to go abroad to give a lecture on IUT theory. The author argues that Mochizuki's IUT theory is radically new even in comparison with Wiles' solution of Fermat's last theorem, so that it is impossible to explicate the theory by such a lecture. I know well that such grandiosity often accomapanies paranoia. Such a claim reminds me of Zen Buddhism, in which it is claimed that nirvana is unspeakable, so that you should do only Zazen for years or decades to attain nirvana. Daisetz Suzuki (1870–1966) is famous for propagating Zen Buddhism worldwide by speaking and writing a lot about Zen Buddhism in English. Some cynics say that Daisetz Suzuki became famous by speaking a lot about the unspeakable. The author of this book has written a book for general public on the unspeakable.

7. I close this review with my favorite song [`https://www.youtube.com/watch?v=HZpVIi3IWhs`]. Je suis un peu fatigu 30fb et je suppose que le lecteur l'est aussi.

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MSC:

14-02	Research exposition (monographs, survey articles) pertaining to algebraic geometry
14-03	History of algebraic geometry
11-02	Research exposition (monographs, survey articles) pertaining to number theory
11-03	History of number theory
11Gxx	Arithmetic algebraic geometry (Diophantine geometry)
14H30	Coverings of curves, fundamental group
01A61	History of mathematics in the 21st century